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String breaking in QCD: dual superconductor vs. stochastic vacuum model

Dmitri Antonov*
Institute of Physics, Humboldt University of Berlin
Newtonstr. 15, 12489 Berlin, Germany
E-mail: antonov@physik.hu-berlin.de

Adriano Di Giacomo
Dipartimento di Fisica "E. Fermi" dell'Universitá di Pisa
INFN-Sezione di Pisa
Largo Pontecorvo, 3, 56127 Pisa, Italy
E-mail: digiaco@df.unipi.it

Abstract: Effects of dispersion of the chromoelectric field of the flux tube on the string-breaking distance are studied. The leading-order correction is shown to slightly diminish the result following from the Schwinger formula. Instead, accounting for corrections of all orders might result, at certain values of the Landau-Ginzburg parameter, in an increase of the string-breaking distance up to one order of magnitude. An alternative formula for this distance is obtained when produced pairs are treated as holes in a confining pellicle, which spans over the contour of an external quark-antiquark pair. Generalizations of the obtained results to the cases of small temperatures, as well as temperatures close to the critical one are also discussed.

Keywords: Nonperturbative Effects, Confinement, Lattice Gauge Field Theories, Phenomenological Models.

*Permanent address: ITEP, B. Cheremushkinskaya 25, RU-117 218 Moscow, Russia.

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Contents

1. Introduction. The model and some relations between its parameters

String breaking at zero and finite temperatures is known as one of the important open problems in QCD (see e.g. refs. [1]–[7] for recent developments). The aim of this paper is to proceed further with the calculation of the string-breaking distance, along with the lines of ref. [6] (sections 2–6), as well as using some alternative model (section 7). In ref. [6], dual Abrikosov-Nielsen-Olesen strings have been used to model flux tubes of the chromoelectric field in real QCD. Such strings [8] are solutions to the classical equations of motion in the 4d dual abelian Higgs model, whose euclidean lagrangian reads $\mathcal{L} = \frac{1}{4}F_{\mu\nu}^2 + |D_\mu\varphi|^2 + \frac{\lambda}{2}(|\varphi|^2 - v^2)^2$. Here $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $D_\mu\varphi = (\partial_\mu - ig_m B_\mu)\varphi$, B_μ is the dual gauge field, φ is the complex-valued dual-Higgs field, and g_m is the magnetic coupling constant, related to the electric one, g, as $g_m = 2\pi/g$. The masses of the dual vector boson and the dual Higgs boson are $m_V = \sqrt{2}g_m v$ and $m_H = \sqrt{2\lambda}v$, respectively.

We will consider this model in the London limit,[1] $L \equiv \ln\frac{m_H}{m_V} \gg 1$, where all the results can be obtained analytically. The Bogomol'nyi case, $L = 0$, will be considered elsewhere. The electric field of a straight-line dual Abrikosov-Nielsen-Olesen string reads

[1]The ratio $\frac{m_H}{m_V}$ is called Landau-Ginzburg parameter.

$E(r) = \frac{m_V^2}{g_m} K_0(m_V r)$, where $r = |\mathbf{x}_\perp|$ with $\mathbf{x}_\perp = (x_1, x_2)$ denoting the direction transverse to the string, and from now on K_ν's stand for MacDonald functions. The field averaged over the string cross section, $\langle E \rangle = \frac{1}{S} \int d^2 r E(r)$, obeys the relation $g\langle E \rangle = 4\sigma/L$. Here $S = \pi m_V^{-2}$ is the area of the cross section of the string, and the string tension has the form $\sigma = 2\pi v^2 L$. In order to establish a correspondence to QCD, we will need to express all the results in terms of σ and g. Two formulae useful for this are $m_V^2 = \frac{4\pi\sigma}{g^2 L}$ and $S = \frac{g^2 L}{4\sigma}$.

2. The string-breaking distance with the neglection of dispersion of $E(r)$

The rate of the pair production reads $w = \frac{2}{S} \operatorname{Im} \Gamma[A_i]$, where the one-loop effective action of a scalar quark has the form:

$$\Gamma[A_i] = N_c N_f \int_0^\infty \frac{dT}{T} \mathrm{e}^{-m^2 T} \times$$
$$\times \int \mathcal{D}\mathbf{x}_\perp \mathcal{D}\mathbf{x}_\parallel \exp\left[- \int_0^T d\tau \left(\frac{1}{4}\dot{\mathbf{x}}_\perp^2 + \frac{1}{4}\dot{\mathbf{x}}_\parallel^2 - \frac{ig}{2} E(\mathbf{x}_\perp(\tau)) \epsilon_{ij} \dot{x}_i x_j \right) \right]. \quad (2.1)$$

Here, $\mathbf{x}_\parallel = (x_3, x_4)$, $i, j = 3, 4$, and the field of the string is $A_i = -\frac{1}{2}\epsilon_{ij} x_j E(\mathbf{x}_\perp)$. Note that, in this paper, we will study the case of scalar quarks only (except of some comment at the end of the next section), since the longitudinal part of the spin factor, $\int \mathcal{D}\psi_i \exp[-\int_0^T d\tau (\frac{1}{2}\psi_i \dot{\psi}_i - ig\psi_i \psi_j F_{ij}(\tau))]$, where $F_{34}(\tau) = -F_{43}(\tau) = E(\mathbf{x}_\perp(\tau))$, cannot be calculated exactly as long as F_{ij} is τ-dependent. To evaluate the path integral (2.1), we will use the requirement that the mass of the produced pair, m, should be much larger than m_V, i.e. $m \gg \frac{2}{g}\sqrt{\frac{\pi\sigma}{L}}$, in order that the field of a string can be considered as a constant one. The "largeness" of m means that characteristic proper times are "small", $T < \frac{1}{m^2}$, that enables us to evaluate the path integral semiclassically and to compute further the leading quantum correction using the Feynman variational method [9]. Furthermore, we naturally assume that not only the Compton wavelength of a produced pair, m^{-1}, is much smaller than the range of the field localization, m_V^{-1}, but also that the characteristic pair trajectories are small compared to m_V^{-1}. Since, in the euclidean space-time, pair trajectories are circles of the radius $R_p = \frac{m}{g\langle E \rangle}$,[2] the condition of smallness of the pair trajectory, $R_p \ll m_V^{-1}$, yields $m \ll 2g\sqrt{\frac{\sigma}{\pi L}}$. Both conditions,

$$\frac{2}{g}\sqrt{\frac{\pi\sigma}{L}} \ll m \ll 2g\sqrt{\frac{\sigma}{\pi L}}\,, \quad (2.2)$$

are compatible to each other at $g \gg 1$.

Then, due to the smallness of a pair trajectory, $E(\mathbf{x}_\perp(\tau))$ can be replaced by its value averaged along the trajectory:

$$\int_0^T d\tau E(\mathbf{x}_\perp(\tau)) \dot{x}_i x_j \simeq -\Sigma_{ij} \cdot \frac{1}{T} \int_0^T d\tau E(\mathbf{x}_\perp(\tau))\,, \quad (2.3)$$

[2] This can be seen either by solving the respective Euler-Lagrange equation [10], or simply by noticing that $\exp(-\frac{\pi m^2}{g\langle E \rangle})$ in the Schwinger formula should be $\mathrm{e}^{-\Phi}$, where Φ is the flux of the electric field through the contour of a pair, $\Phi = g\langle E \rangle \cdot \pi R_p^2$. Therefore, a pair is identified with a quark which moves along a circle of the radius R_p.

where $\Sigma_{ij} \equiv \int_0^T d\tau x_i \dot{x}_j$ is the (i,j)-th component of the so-called tensor area of the trajectory. In the leading small-T approximation, we obtain the classical result for $\int \mathcal{D}\mathbf{x}_\perp = \int d^2x_\perp(0) \int_{\mathbf{x}_\perp(0)=\mathbf{x}_\perp(T)} \mathcal{D}\mathbf{x}_\perp(\tau)$ in eq. (2.1):

$$\frac{1}{4\pi T}\int d^2x_\perp \exp\left[-\frac{ig}{2}E(\mathbf{x}_\perp)\epsilon_{ij}\Sigma_{ij}\right]. \tag{2.4}$$

Thus,

$$\Gamma[A_i] \simeq \frac{N_cN_f}{4\pi}\int_0^\infty \frac{dT}{T^2}\mathrm{e}^{-m^2T}\int \mathcal{D}\mathbf{x}_\parallel \exp\left(-\frac{1}{4}\int_0^T d\tau \dot{\mathbf{x}}_\parallel^2\right)\int d^2x_\perp \exp\left[-\frac{ig}{2}E(\mathbf{x}_\perp)\epsilon_{ij}\Sigma_{ij}\right]. \tag{2.5}$$

Neglecting for this section the dispersion of the field $E(\mathbf{x}_\perp)$, we have

$$\int d^2x_\perp \exp\left[-\frac{ig}{2}E(\mathbf{x}_\perp)\epsilon_{ij}\Sigma_{ij}\right] \equiv S\left\langle \exp\left[-\frac{ig}{2}E(\mathbf{x}_\perp)\epsilon_{ij}\Sigma_{ij}\right]\right\rangle \simeq S\exp\left[-\frac{ig}{2}\langle E\rangle\epsilon_{ij}\Sigma_{ij}\right]. \tag{2.6}$$

In this approximation, we therefore arrive at the Euler-Heisenberg lagrangian in the constant field $\bar{A}_i \equiv -\frac{1}{2}\epsilon_{ij}x_j\langle E\rangle$,

$$\Gamma[A_i] \simeq S\frac{N_cN_f}{(4\pi)^2}\int_0^\infty \frac{dT}{T^2}\mathrm{e}^{-m^2T}\frac{g\langle E\rangle}{\sin(g\langle E\rangle T)}, \tag{2.7}$$

and recover for w the 4d Schwinger result[3] in the bosonic case:

$$w = N_cN_f\frac{(g\langle E\rangle)^2}{(2\pi)^3}\sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^2}\exp\left(-\frac{\pi k m^2}{g\langle E\rangle}\right). \tag{2.8}$$

We can further express the inequality $T < \frac{1}{m^2}$ in terms of the parameters of our model. Namely, since at least the first pole in the imaginary part of the Euler-Heisenberg lagrangian should give its contribution, T may not be arbitrarily small, but the following inequality should rather hold: $T > \frac{\pi}{g\langle E\rangle} = \frac{\pi L}{4\sigma}$. The condition $\frac{1}{m^2} > T$ then yields $m < 2\sqrt{\frac{\sigma}{\pi L}}$, that also coincides with the condition for w not to be exponentially small. This new constraint is stronger than the above-obtained one, expressed by the right inequality of (2.2), since $g \gg 1$ is now absent. The new constraint can be viewed as an upper boundary on L:

$$L < \frac{4}{\pi}\frac{\sigma}{m^2} \simeq 1.27\frac{\sigma}{m^2}. \tag{2.9}$$

Setting $\sigma = (440\,\mathrm{MeV})^2$ and a typical hadronic mass $m = 200\,\mathrm{MeV}$, we get an estimate $L < 6.2$, that still leaves a window for $L \gg 1$.

Approximating the whole sum (2.8) by its first term (equal to the density of produced pairs), we have

$$w \simeq \frac{2N_cN_f}{\pi^3}\left(\frac{\sigma}{L}\right)^2\exp\left(-\frac{\pi m^2 L}{4\sigma}\right). \tag{2.10}$$

The respective string-breaking distance [6] $\bar{r} = \frac{1}{\sqrt{2Sw}}$ has the form

$$\bar{r} = \frac{\pi^{3/2}\sqrt{L}}{g\sqrt{N_cN_f\sigma}}\exp\left(\frac{\pi L}{8}\frac{m^2}{\sigma}\right), \tag{2.11}$$

where, due to inequality (2.9), $\exp(\frac{\pi L}{8}\frac{m^2}{\sigma}) < \sqrt{\mathrm{e}} \simeq 1.65$.

[3] Up to the factor N_cN_f absent in the electromagnetic case.

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3. Finite-temperature generalizations

Setting for the mass of a produced pair the π-meson mass, we can further extend the analysis of the previous section to the case where the temperature is close to the critical one. This can be done by using the formulae [6, 11] $m^2 = m_\pi^2 t^{1.44}$, $\sigma = \sigma_0 t^{0.33}$, where $t \equiv 1 - \frac{T}{T_c}$ is the reduced temperature [4], and $\sigma_0 \simeq (440\,\mathrm{MeV})^2$. We obtain:

$$w \to \frac{2N_c N_f}{\pi^3}\left(\frac{\sigma}{L}\right)^2 = \mathcal{O}\left(t^{0.66}\right), \tag{3.1}$$

that establishes the law by which $\bar{r}$ grows at $t \to 0$. In the same limit $t \to 0$, the condition $mr_\perp \gg 1$ with the temperature-dependent $r_\perp$, $r_\perp = \frac{g}{2}\sqrt{\frac{L}{\pi\sigma}}$, leads to the following boundary on t from below: $\frac{1}{L}\left(\frac{1}{m_\pi}\sqrt{\frac{\sigma_0}{\alpha_s}}\right)^{1.79} \ll t$. On the other hand, the condition of smallness of the pair trajectory, expressed by the right inequality of (2.2), sets a boundary on t from above: $t \ll \frac{1}{L^{0.91}}\left(\frac{2g}{m_\pi}\sqrt{\frac{\sigma_0}{\pi}}\right)^{1.82}$. These two conditions imposed on t are apparently compatible to each other at sufficiently large L and/or g. Note also that the condition (2.9) becomes softer as one approaches the critical point, since $\frac{\sigma}{m^2} = \frac{\sigma_0}{m_\pi^2} t^{-1.11} \to \infty$.

Let us now evaluate the string-breaking distance at relatively low temperatures, namely at $T = \mathcal{O}(f_\pi)$, where the following formula holds [12]:

$$m_\pi^2(T) \simeq m_\pi^2\left(1 + \frac{T^2}{24 f_\pi^2}\right). \tag{3.2}$$

We can further approximate $m_V(T)$ by $m_V(0)$, since m_V^{-1} is the vacuum correlation length, whose QCD-analogue at such temperatures can be considered as temperature-independent [13].[5]

Next, we need to know the temperature dependence of $g\langle E\rangle$. Since we are exploring the region of temperatures smaller than the temperature of dimensional reduction, the sum over Matsubara frequencies appears:

$$g\left\langle E\right\rangle(T) = \frac{(gm_V)^2}{2\pi^2}\int d^2z \sum_{n=-\infty}^{+\infty} K_0\left(\sqrt{z^2 + (m_V\beta n)^2}\right),$$

where $\beta \equiv 1/T$. To carry out the integral, it is convenient to transform the sum as follows:

$$\begin{aligned}\sum_{n=-\infty}^{+\infty} K_0\left(\sqrt{z^2+(m_V\beta n)^2}\right) &= \frac{1}{2}\sum_{n=-\infty}^{+\infty}\int_0^\infty \frac{dt}{t}\exp\left[-\frac{1}{4t} - t\left(z^2 + (m_V\beta n)^2\right)\right] \\ &= \frac{T\sqrt{\pi}}{2m_V}\sum_{n=-\infty}^{+\infty}\int_0^\infty \frac{dt}{t^{3/2}}\exp\left[-z^2 t - \frac{1+(\omega_n/m_V)^2}{4t}\right] \\ &= \frac{\pi T}{m_V}\sum_{n=-\infty}^{+\infty}\frac{\mathrm{e}^{-|z|\sqrt{1+(\omega_n/m_V)^2}}}{\sqrt{1+(\omega_n/m_V)^2}}, \end{aligned} \tag{3.3}$$

[4]The fact that the temperature and the proper time are denoted by the same letter "T" should not lead to reader's confusions.

[5]In QCD, the (magnetic) vacuum correlation length becomes definitely temperature-dependent at temperatures larger than the temperature of dimensional reduction (that is of the order of $2T_c$), where it is proportional to $\frac{1}{g^2(T)T}$.

where $\omega_n \equiv 2\pi T n$. The integration over d^2z then immediately yields

$$g\langle E\rangle(T) = g^2 m_V T \sum_{n=-\infty}^{+\infty} \frac{1}{[1+(\omega_n/m_V)^2]^{3/2}} . \tag{3.4}$$

To study the zero-temperature limit of this expression, one should perform the inverse transformation of the sum:

$$\begin{aligned}
\sum_{n=-\infty}^{+\infty} \frac{1}{[1+(\omega_n/m_V)^2]^{3/2}} &= \frac{2}{\sqrt{\pi}} \int_0^\infty dt \sqrt{t} \mathrm{e}^{-t} \sum_{n=-\infty}^{+\infty} \mathrm{e}^{-t(\omega_n/m_V)^2} \\
&= \frac{m_V \beta}{\pi} \sum_{n=-\infty}^{+\infty} \int_0^\infty dt \exp\left[-t - \frac{(m_V\beta n)^2}{4t}\right] \\
&= \frac{(m_V\beta)^2}{\pi} \sum_{n=-\infty}^{+\infty} |n| K_1(m_V\beta|n|) .
\end{aligned}$$

At small T's of interest, the sum here is apparently dominated by the zeroth mode. Moreover, expanding the function K_1 in power series, one can check that no finite corrections exist to the r.h.s. of the formula $|n|K_1(m_V\beta|n|) \stackrel{|n|\to 0}{\longrightarrow} T/m_V$. Therefore, the leading small-T (physically, at $T \ll f_\pi$) correction to $g\langle E\rangle(T)$ stems from the terms in the sum with $|n| = 1$, $2K_1(m_V\beta)$. This correction is therefore exponentially small, as confirmed by the following final expression:

$$g\langle E\rangle(T) \simeq \frac{4\sigma_0}{L}\left[1 + 2\sqrt{\frac{\beta}{g}}\left(\frac{\pi^3\sigma_0}{L}\right)^{1/4} \mathrm{e}^{-\frac{2}{g}\sqrt{\frac{\pi\sigma_0}{L}}\beta}\right], \tag{3.5}$$

where $\sigma_0 \equiv 2\pi v^2 L$. As for the dependence $\sigma(T)$, it is derived in appendix B.

Equations (3.2) and (3.5), being substituted into the formula $w \simeq N_c N_f \frac{(g\langle E\rangle)^2}{(2\pi)^3} \mathrm{e}^{-\frac{\pi m^2}{g\langle E\rangle}}$, determine the temperature dependence of w. Apparently, the correction produced by eq. (3.5) is exponentially small with respect to the temperature dependence appearing by means of eq. (3.2). Therefore, w decreases at T increasing from zero to the temperatures of the order of f_π. The corresponding increase of $\bar{r}$ parallels the same phenomenon we have found at $T \to T_c$.

It is finally worth making the following comment. In the case where the dispersion of $E(r)$ is neglected (that we are discussing here), the spin factor can be evaluated, and it is natural to address the issue of how strongly the antiperiodic boundary conditions for spin-$\frac{1}{2}$ quarks can affect the obtained result. For such quarks, one should substitute in eq. (2.7) $\frac{1}{\sin(g\langle E\rangle T)} \to -\frac{2}{\tan(g\langle E\rangle T)}$, that yields, instead of eq. (2.8),

$$w = N_c N_f \frac{(g\langle E\rangle)^2}{4\pi^3} \sum_{k=1}^{\infty} \frac{1}{k^2} \exp\left(-\frac{\pi k m^2}{g\langle E\rangle}\right). \tag{3.6}$$

It has been shown in ref. [14] that antiperiodic boundary conditions for fermions can be taken into account upon the multiplication of the zero-temperature one-loop effective action by the factor $[1 + 2\sum_{n=1}^{\infty}(-1)^n \mathrm{e}^{-\frac{\beta^2 n^2}{4T}}]$, where (only for this formula in this section) T stands for the proper time. In course of taking $\mathrm{Im}\,\Gamma[A_i]$, this factor transforms to another one, by which a k-th term of the series in eq. (3.6) should be multiplied: $[1+2\sum_{n=1}^{\infty}(-1)^n\exp(-\frac{g\langle E\rangle\beta^2n^2}{4\pi k})]$. For relevant k's, which are $k<\frac{g\langle E\rangle}{\pi m^2}$, we obtain the following relevant n's: $n<2T\sqrt{\frac{\pi k}{g\langle E\rangle}}=\frac{2T}{m}$, that, at $T<f_\pi$, are smaller than 2. Therefore, the factor produced by the antiperiodic boundary conditions for spin-$\frac{1}{2}$ quarks reduces to $[1-2\exp(-\frac{g\langle E\rangle\beta^2}{4\pi k})]$. Since, for the above-mentioned relevant k's, $\frac{g\langle E\rangle\beta^2}{4\pi k}>(\frac{m\beta}{2})^2$, at $T<\frac{m_\pi}{\sqrt{2}}$ the obtained correction falls off as the tail of the gaussian distribution.[6]

4. Significance of corrections due to the dispersion of $E(r)$

Let us now take into account the second cumulant in eq. (2.6) (i.e. the dispersion of $E(r)$) and also address the issue of convergence of the cumulant expansion. We have

$$\left\langle\exp\left[-\frac{ig}{2}E(\mathbf{x}_\perp)\epsilon_{ij}\Sigma_{ij}\right]\right\rangle\simeq\exp\left[-\frac{ig}{2}\langle E\rangle\,\epsilon_{ij}\Sigma_{ij}-\frac{g^2}{8}(\epsilon_{ij}\Sigma_{ij})^2\left(\langle E^2\rangle-\langle E\rangle^2\right)\right]$$
$$=\frac{1}{2\sqrt{\pi c}}\int_{-\infty}^{+\infty}df\,\exp\left[-\frac{f^2}{4c}+i\epsilon_{ij}\Sigma_{ij}\left(f-\frac{g}{2}\langle E\rangle\right)\right], \tag{4.1}$$

where

$$c\equiv\frac{g^2}{8}\left(\langle E^2\rangle-\langle E\rangle^2\right). \tag{4.2}$$

The last formula in (4.1) means that, instead of the constant field $\langle E\rangle$, we are now dealing with a shifted (but still space-independent) one $\bar{E}\equiv\langle E\rangle-\frac{2f}{g}$, where the field f should be eventually averaged over. The respective pair-production rate reads

$$w=\frac{N_cN_f}{2\sqrt{\pi c}}\int_{-\infty}^{+\infty}df\mathrm{e}^{-\frac{f^2}{4c}}\cdot\frac{(g\bar{E})^2}{(2\pi)^3}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}\exp\left(-\frac{\pi km^2}{g\bar{E}}\right). \tag{4.3}$$

The parameter of the cumulant expansion, i.e. the ratio of the absolute value of the second cumulant to that of the first one, is

$$\frac{g}{2}\cdot 2\pi R_p^2\cdot\frac{\langle E^2\rangle-\langle E\rangle^2}{\langle E\rangle}=g\,\langle E\rangle\cdot\pi R_p^2 d. \tag{4.4}$$

Here,

$$d\equiv\frac{\langle E^2\rangle-\langle E\rangle^2}{\langle E\rangle^2} \tag{4.5}$$

is a measure of dispersion of the field $E(r)$. It is calculated in appendix A and reads $d=\frac{\pi}{2}-1$. The condition of convergence of the cumulant expansion, i.e. the demand that the parameter (4.4) is smaller than unity, then yields yet another upper boundary on L:

$$L<\frac{4}{\pi d}\frac{\sigma}{m^2}\simeq 2.23\frac{\sigma}{m^2}, \tag{4.6}$$

[6]The inequality $T<\frac{m_\pi}{\sqrt{2}}$ stems from the fact that the width of the gaussian distribution $\mathrm{e}^{-m^2\beta^2/4}$ is $\frac{\sqrt{2}}{m}$.

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that is, however, weaker than the condition (2.9). As well as eq. (2.9), this new condition on L becomes softer at $T \to T_c$, since $\frac{\sigma}{m^2} = \frac{\sigma_0}{m_\pi^2} t^{-1.11} \to \infty$. In another words, at a fixed L, obeying condition (4.6), cumulant expansion converges better at $T \to T_c$, since its parameter vanishes in this limit as $\mathcal{O}(t^{1.11})$.

To perform the average over f in eq. (4.3), note that the dispersion (i.e. the width) of the gaussian distribution, e^{-ax^2}, is $\frac{1}{\sqrt{2a}}$. Therefore, characteristic f's obey the estimate $|f| \le \sqrt{2c} = \frac{g}{2}\sqrt{\langle E^2\rangle - \langle E\rangle^2}$, and $\frac{2|f|/g}{\langle E\rangle} \le \sqrt{d} \simeq \sqrt{0.57} \simeq 0.76$. Therefore, although with the accuracy of only 76% (cf. section 6, where this problem will be avoided), we can approximately write

$$\frac{1}{E} \simeq \frac{1}{\langle E\rangle}\left(1 + \frac{2f}{g\langle E\rangle}\right). \tag{4.7}$$

When this expansion is substituted into eq. (4.3), the f-integration can already be performed and yields (cf. eq. (2.8))

$$w \simeq N_c N_f \frac{(g\langle E\rangle)^2}{(2\pi)^3} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}\left[1 + \frac{2\pi m^2 k}{g\langle E\rangle} d\right] \exp\left[-\frac{\pi m^2 k}{g\langle E\rangle}\left(1 - \frac{\pi m^2 k}{2g\langle E\rangle} d\right)\right]. \tag{4.8}$$

Approximating again the whole sum by the first term only, we see that the obtained correction is small, provided $\frac{\pi m^2}{g\langle E\rangle} d \ll 1$. This is precisely the condition of convergence of the cumulant expansion, eq. (4.6), with "$<$" replaced by "$\ll$". In particular, the correction produced by the second cumulant becomes vanishingly smaller than the leading term at $T \to T_c$. We also see that the obtained correction increases w, thus diminishing $\bar{r}$. Due to condition (2.9), $\bar{r}$ diminishes only by a factor of the order of unity.

5. Accounting for quantum effects by the Feynman variational method

In this section, we will evaluate the leading quantum correction to eq. (4.8). It can be obtained upon the small-T analysis of the path integral over $\mathbf{x}_\perp(\tau)$ in eq. (2.1) with the approximation (2.3) adopted. Such an integral can naturally be evaluated using the Feynman variational method [9]. In 2d-case and in our notations, it looks as follows. We need to evaluate the path integral

$$\mathcal{Z}(T) \equiv \int d^2x(0) \int_{\mathbf{x}(0)=\mathbf{x}(T)} \mathcal{D}\mathbf{x}(\tau)\mathrm{e}^{-\mathcal{S}},$$

where $\mathcal{S} \equiv \int_0^T d\tau\left(\frac{\dot{\mathbf{x}}^2}{4} + U(\mathbf{x})\right)$, $U(\mathbf{x}) \equiv \frac{ig}{2T} E(\mathbf{x})\epsilon_{ij}\Sigma_{ij}$. The classical expression for this integral, $\frac{1}{4\pi T}\int d^2x\mathrm{e}^{-TU(\mathbf{x})}$, given by eq. (2.4), corresponds to T's, which are so small that the trajectory does not deviate from its initial point $\mathbf{x}(0)$. Let us further introduce the coordinate describing the position of the trajectory $\mathbf{x}_0 = \frac{1}{T}\int_0^T d\tau \mathbf{x}(\tau)$, the trial action $\mathcal{S}_0 = \int_0^T d\tau \frac{\dot{\mathbf{x}}^2}{4} + TW(\mathbf{x}_0)$, and the respective trial partition function

$$\begin{aligned}\mathcal{Z}_0(T) &= \int d^2x_0 \int_{\text{fixed } \mathbf{x}_0} \mathcal{D}\mathbf{x}\mathrm{e}^{-\mathcal{S}_0} \\ &= \int d^2x_0 \int_{\mathbf{y}(0)=\mathbf{y}(T)=0} \mathcal{D}\mathbf{y}\mathrm{e}^{-\int_0^T d\tau \frac{\dot{\mathbf{y}}^2}{4} - TW(\mathbf{x}_0)} = \frac{1}{4\pi T}\int d^2x\mathrm{e}^{-TW(\mathbf{x})},\end{aligned} \tag{5.1}$$

where $\mathbf{y}(\tau) = \mathbf{x}(\tau) - \mathbf{x}(0)$. Here W is a trial function, which should be determined upon the minimization of the expression $F_0 + \frac{1}{T}\langle \mathcal{S} - \mathcal{S}_0\rangle_{\mathcal{S}_0}$ (which approximates the true free energy F of the system from the above, $F \le F_0 + \frac{1}{T}\langle \mathcal{S} - \mathcal{S}_0\rangle_{\mathcal{S}_0}$), where $F = -\frac{1}{T}\ln \mathcal{Z}(T)$, $F_0 = -\frac{1}{T}\ln \mathcal{Z}_0(T)$. It is further possible to demonstrate that the averaged difference of actions can be written as

$$\frac{1}{T}\left\langle \mathcal{S} - \mathcal{S}_0\right\rangle_{\mathcal{S}_0} = \left\langle U(\mathbf{x}(0))\right\rangle_{\mathcal{S}_0} - \left\langle W(\mathbf{x}_0)\right\rangle_{\mathcal{S}_0}.$$

For the first of the two averages here one has $\langle U(\mathbf{x}(0))\rangle_{\mathcal{S}_0} = \int d^2y \mathcal{K}(\mathbf{y}) \mathrm{e}^{-TW(\mathbf{y})}$, where

$$\begin{aligned}\mathcal{K}(\mathbf{y}) &\equiv \int_{\mathbf{x}(0)=\mathbf{x}(T)} \mathcal{D}\mathbf{x} U(\mathbf{x}(0)) \exp\left(-\int_0^T d\tau \frac{\dot{\mathbf{x}}^2}{4}\right)\delta(\mathbf{y} - \mathbf{x}_0) \\ &= \int \frac{d^2q}{(2\pi)^2}\tilde{U}(\mathbf{q}) \int \frac{d^2k}{(2\pi)^2} \mathrm{e}^{-i\mathbf{k}\mathbf{y}} \times \\ &\times \int_{\mathbf{x}(0)=\mathbf{x}(T)} \mathcal{D}\mathbf{x} \exp\left[-\int_0^T d\tau \frac{\dot{\mathbf{x}}^2}{4} + \frac{i}{T}\mathbf{k}\int_0^T d\tau \mathbf{x} + i\mathbf{q}\mathbf{x}(0)\right],\end{aligned}$$

and $\tilde{U}(\mathbf{q}) \equiv \int d^2x \mathrm{e}^{-i\mathbf{q}\mathbf{x}} U(\mathbf{x})$ is the Fourier image of $U(\mathbf{x})$. Using the formula

$$\begin{aligned}&\int_{\mathbf{x}(0)=\mathbf{x}(T)} \mathcal{D}\mathbf{x} \exp\left[-\int_0^T d\tau \left(\frac{\dot{\mathbf{x}}^2}{4} + i\mathbf{f}\mathbf{x}\right)\right] = \\ &= \frac{\pi}{T}\delta\left(\int_0^T d\tau \mathbf{f}\right) \exp\left[\frac{1}{2}\int_0^T d\tau \int_0^T d\tau' |\tau - \tau'| \mathbf{f}(\tau)\mathbf{f}(\tau') + \frac{1}{T}\sum_{\alpha=1}^{2}\left(\int_0^T d\tau \tau f_\alpha\right)^2\right]\end{aligned}$$

with $\mathbf{f} = \frac{\mathbf{k}}{T} + \mathbf{q}\delta(\tau)$, one obtains $\mathcal{K}(\mathbf{y}) = \frac{3}{(2\pi T)^2}\int d^2x U(\mathbf{x}) \mathrm{e}^{-\frac{3}{T}(\mathbf{x}-\mathbf{y})^2}$. Thus, according to eq. (5.1),

$$\frac{1}{T}\left\langle \mathcal{S} - \mathcal{S}_0\right\rangle_{\mathcal{S}_0} = \frac{\int d^2y \mathrm{e}^{-TW(\mathbf{y})}\left[4\pi T\mathcal{K}(\mathbf{y}) - W(\mathbf{y})\right]}{\int d^2y \mathrm{e}^{-TW(\mathbf{y})}}.$$

Note that the potential smeared over the gaussian distribution, $4\pi T\mathcal{K}(\mathbf{y})$, goes over to the unsmeared potential $U(\mathbf{y})$ in the limit $T \to 0$, as it should be.

The variational equation $\delta[F_0 + \frac{1}{T}\langle \mathcal{S} - \mathcal{S}_0\rangle_{\mathcal{S}_0}] = 0$ then yields $W(\mathbf{y}) = 4\pi T\mathcal{K}(\mathbf{y})$ as the best choice of $W(\mathbf{y})$.[7]. Accordingly, the new expression, which accounts for quantum effects,

$$\mathcal{Z}_0(T) = \frac{1}{4\pi T}\int d^2y \mathrm{e}^{-4\pi T\mathcal{K}(\mathbf{y})} = \frac{1}{4\pi T}\int d^2y \exp\left[-\frac{3}{\pi}\int d^2x U(\mathbf{x}) \mathrm{e}^{-\frac{3}{T}(\mathbf{x}-\mathbf{y})^2}\right] \tag{5.2}$$

is a better approximation to $\mathcal{Z}(T)$ than its purely classical counterpart $\frac{1}{4\pi T}\int d^2x \mathrm{e}^{-TU(\mathbf{x})}$ (recoverable in the limit $T \to 0$), which was used before.

The obtained result (5.2) prescribes to replace $E(\mathbf{x}_\perp)$ in eq. (2.6) by

$$\mathcal{E}(\mathbf{x}_\perp) \equiv \frac{3}{\pi T}\int d^2y E(\mathbf{y}) \mathrm{e}^{-\frac{3}{T}(\mathbf{y}-\mathbf{x}_\perp)^2}.$$

[7]With this choice of $W(\mathbf{y})$, $\langle \mathcal{S} - \mathcal{S}_0\rangle_{\mathcal{S}_0} = 0$.

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In particular, $\langle \mathcal{E} \rangle = \langle E \rangle$, i.e., at the level of the first cumulant, the variational method yields the same result as the classical approximation. We further obtain

$$\langle \mathcal{E}^2 \rangle = \frac{3}{2\pi TS} \int d^2x d^2y \mathrm{e}^{-\frac{3}{2T}(\mathbf{x}-\mathbf{y})^2} E(\mathbf{x}) E(\mathbf{y})\,.$$

This expression replaces $\langle E^2 \rangle$ in eq. (4.1); in particular, $\langle \mathcal{E}^2 \rangle \xrightarrow{T\to 0} \langle E^2 \rangle$. Thus, within the variational approach, the constant (4.2) becomes replaced by $c_{\rm var} \equiv \frac{g^2}{8}(\langle \mathcal{E}^2 \rangle - \langle E \rangle^2)$. An apparent difference of $c_{\rm var}$ from c is that the former is T-dependent, whereas the latter is not.[8] The one-loop effective action reads

$$\Gamma[A_i] \simeq S\frac{N_c N_f}{32\pi^{5/2}} \int_0^\infty \frac{dT}{T^2} \mathrm{e}^{-m^2T} \frac{1}{\sqrt{c_{\rm var}}} \int_{-\infty}^{+\infty} df \mathrm{e}^{-\frac{f^2}{4c_{\rm var}}} \frac{g\bar{E}}{\sin(g\bar{E}T)}\,,$$

where "$\simeq$" means "in the bilocal approximation to the cumulant expansion", and again $\bar{E} \equiv \langle E \rangle - \frac{2f}{g}$.

Let us further calculate the integral entering $c_{\rm var}$

$$\int d^2x d^2y \mathrm{e}^{-\frac{3}{2T}(\mathbf{x}-\mathbf{y})^2} K_0(m_V|\mathbf{x}|) K_0(m_V|\mathbf{y}|) = \frac{1}{m_V^4} \int d^2z d^2z' \mathrm{e}^{-\frac{3}{2Tm_V^2}(\mathbf{z}-\mathbf{z}')^2} K_0(z) K_0(z')\,, \tag{5.3}$$

where $z \equiv |\mathbf{z}|$, $z' \equiv |\mathbf{z}'|$. This can be done approximately, by using the fact that $T < \frac{1}{m^2} \ll \frac{1}{m_V^2}$, i.e. $\frac{1}{Tm_V^2} \gg 1$, that allows us to Taylor expand $K_0(z')$ around $\mathbf{z}$. This calculation, whose details are given in appendix A, leads to the following result:

$$c_{\rm var} = c + \frac{\sigma^3}{6g^2L^3} \frac{y}{g\bar{E}}\,, \tag{5.4}$$

where the variable $y \equiv g\bar{E}T$ acquires the values πk when one is taking $\mathrm{Im}\,\Gamma[A_i]$. We can further use the approximation (4.7) with the term $\frac{2f}{g\langle E \rangle}$ neglected, since it would otherwise lead to the excess of accuracy, making the f-integral non-gaussian. Denoting this approximation by "$\simeq$" and substituting the above-mentioned values of y, we obtain the following modification of eq. (4.2):

$$c_{\rm var} \simeq \frac{g^2}{8} \left[\left(1 + \frac{\pi k}{48 g^2}\right) \langle E^2 \rangle - \langle E \rangle^2 \right].$$

This finally results in the following change of the dispersion parameter (4.5), entering eq. (4.8), that makes this parameter k-dependent:

$$d \to d_k \equiv \frac{\left(1 + \frac{\pi k}{48g^2}\right) \langle E^2 \rangle - \langle E \rangle^2}{\langle E \rangle^2}\,.$$

This formula describes the quantum correction to eq. (4.8). We, however, see that this correction is small. Indeed, due to the main exponential factor in eq. (4.8), we have for

[8] In particular, this means that the f-integral in eq. (4.1) should now be considered only inside the T-integral (and not vice versa).

relevant k's:

$$\frac{\pi k}{48g^2} < \frac{\pi}{48g^2} \cdot \frac{g\langle E\rangle}{\pi m^2} = \frac{\sigma}{12L(gm)^2} \ll \frac{1}{48\pi},$$

where the last inequality stems from the condition $m \gg m_V$ (expressed by the left inequality of (2.2)).

6. Accounting for the space dependence of $E(r)$ without the cumulant expansion

In this section, we will evaluate w in an alternative way, namely completely without the use of cumulant expansion in eq. (2.6). The necessity of this calculation is, firstly, because cumulant expansion has been shown to converge only provided L is bounded from above according to the condition (4.6). The new method enables one to replace this constraint by some other, weaker, one, and to relax the constraint (2.9). Secondly, the approximation used to arrive at eq. (4.8), which was holding with only 76% accuracy, will now be not necessary anymore.

The method of this section is based on averaging every term in the expansion of the exponent in eq. (2.6) over $d^2x_\perp$. In the London limit, such an average can be done analytically by making use of an explicit form of $E(r)$. Namely, we have

$$\int d^2x_\perp \exp\left[-\frac{ig}{2}\epsilon_{ij}\Sigma_{ij}E(\mathbf{x}_\perp)\right] = \sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{ig}{2}\epsilon_{ij}\Sigma_{ij}\right)^n\left(\frac{m_V^2}{g_m}\right)^n\int d^2x_\perp (K_0(m_V r))^n.$$

The last integral approximately equals $\frac{\pi}{m_V^2}2^{2-n}n!$, that yields

$$\frac{4\pi}{m_V^2}\sum_{n=0}^{\infty}\left(-\frac{ig}{4}\frac{m_V^2}{g_m}\epsilon_{ij}\Sigma_{ij}\right)^n = \frac{4\pi/m_V^2}{1+\frac{ig^2m_V^2}{8\pi}\epsilon_{ij}\Sigma_{ij}} = \frac{4\pi}{m_V^2}\int_0^\infty dt \mathrm{e}^{-t\left(1+\frac{ig^2m_V^2}{8\pi}\epsilon_{ij}\Sigma_{ij}\right)}.$$

The series above converges at $\frac{g^2m_V^2}{8\pi}\cdot 2\pi R_p^2 < 1$, that results in the following boundary on L from above (cf. eq. (4.6)):

$$L < \frac{16}{\pi}\frac{\sigma}{m^2} \simeq 5.09\frac{\sigma}{m^2}. \tag{6.1}$$

We further obtain for eq. (2.5):

$$\begin{aligned}\Gamma[A_i] &\simeq S\frac{N_cN_f}{\pi}\int_0^\infty\frac{dT}{T^2}\mathrm{e}^{-m^2T}\int\mathcal{D}\mathbf{x}_\parallel\int_0^\infty dt\mathrm{e}^{-t\left(1+\frac{ig^2m_V^2}{8\pi}\epsilon_{ij}\Sigma_{ij}\right)}\\ &= S\frac{N_cN_f}{4\pi^2}\int_0^\infty\frac{dT}{T^2}\mathrm{e}^{-m^2T}\int_0^\infty dt\mathrm{e}^{-t}\frac{g\bar{E}}{\sin(g\bar{E}T)},\end{aligned} \tag{6.2}$$

where $\bar{E} \equiv \frac{tgm_V^2}{4\pi}$ is a new space-independent electric field. We see that the constraint (2.9) is now removed. Indeed, the condition for the first pole in $\mathrm{Im}\,\Gamma[A_i]$ to contribute, $\pi \le g\bar{E}T$, together with the inequality $T < \frac{1}{m^2}$ yield $\pi < \frac{g\bar{E}}{m^2} = \frac{t}{L}\frac{\sigma}{m^2}$. This condition does not produce anymore a constraint on L, but merely means that t's obeying the inequality $t > \pi L\frac{m^2}{\sigma}$ give a dominant contribution to $\mathrm{Im}\,\Gamma[A_i]$.

The pair-creation rate stemming from eq. (6.2) takes the form

$$w = \frac{N_c N_f}{2\pi^3}\left(\frac{\sigma}{L}\right)^2 \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}\int_0^\infty dt t^2 \mathrm{e}^{-t-\frac{\pi L m^2 k}{\sigma t}}$$
$$= N_c N_f \frac{m^3}{\pi^{3/2}}\sqrt{\frac{\sigma}{L}}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\sqrt{k}} K_3\left(2m\sqrt{\frac{\pi L k}{\sigma}}\right).$$

Clearly, only terms with $k \le \frac{\sigma}{4\pi L m^2}$ are relevant in the sum. At $T = 0$, substituting $\sigma = (440\,\mathrm{MeV})^2$ and a typical value $m = 200\,\mathrm{MeV}$, we obtain $k < \frac{1}{L} < 1$. Therefore, only the first term is relevant, that yields

$$w \simeq N_c N_f \frac{m^3}{\pi^{3/2}}\sqrt{\frac{\sigma}{L}} K_3\left(2m\sqrt{\frac{\pi L}{\sigma}}\right). \tag{6.3}$$

According to the above-obtained constraint (6.1), the argument of the MacDonald function in this formula is smaller than 8. For the values of L, at which this argument is still larger than unity, i.e. $L > \frac{\sigma}{4\pi m^2}$,

$$w \simeq N_c N_f \frac{m^{5/2}\sigma^{3/4}}{2\pi^{5/4}L^{3/4}} \mathrm{e}^{-2m\sqrt{\frac{\pi L}{\sigma}}}. \tag{6.4}$$

When L obeys simultaneously inequality (2.9), the obtained expression can be compared to eq. (2.10). The parametric dependences of these two expression on m, σ, and L are apparently different from each other. The regime under discussion, $2m\sqrt{\frac{\pi L}{\sigma}} > 1$, however, does not imply the exponential smallness of (6.4), since, due to (2.9), $2m\sqrt{\frac{\pi L}{\sigma}} < 4$. The string-breaking distance, corresponding to eq. (6.4), reads

$$\bar{r} = \frac{2\pi^{5/8}}{g\sqrt{N_c N_f}}\left(\frac{\sigma}{L}\right)^{1/8}\frac{1}{m^{5/4}}\mathrm{e}^{m\sqrt{\frac{\pi L}{\sigma}}}. \tag{6.5}$$

Its ratio to distance (2.11) is given by the function $\frac{2}{\pi^{1/4}x^{5/4}}\mathrm{e}^{x-\frac{x^2}{8}}$, where the variable $x \equiv m\sqrt{\frac{\pi L}{\sigma}}$ ranges between $\frac{1}{2}$ and 2. This is a monotonically decreasing function, which therefore acquires its maximum at $x = \frac{1}{2}$ (where approximation (6.4) to eq. (6.3) starts breaking down). The value of the maximum, $\simeq 9.10$, is, thus, an upper limit for the ratio of the two string-breaking distances. The minimal value of this ratio, corresponding to $x = 2$, is only $\simeq 2.84$.

Instead, at $T \to T_c$, $\frac{m}{\sqrt{\sigma}} = \frac{m_\pi}{\sqrt{\sigma_0}}t^{0.55} \to 0$, and we have $w \to \frac{N_c N_f}{\pi^3}\left(\frac{\sigma}{L}\right)^2$, that differs from eq. (3.1) only by a factor 2. The respective string-breaking distance is, therefore, larger only by a factor $\sqrt{2}$.

7. Considering pairs as holes in the confining pellicle

In this section, we will consider an alternative approach to the pair production, based on a combination of the formulae on the metastable vacuum decay [15, 16] with the stochastic vacuum model [17] (for a recent review see [18]). The idea is to consider the produced pairs as holes in a 2d confining pellicle, which spans over the contour of an external $q\bar{q}$-pair. Such a hole is a region where the pellicle is eaten up. Therefore, a hole of a radius R diminishes the action of the pellicle by $\sigma \cdot \pi R^2$, but increases this action by $(m+\mu)\cdot 2\pi R$. Here μ is some parameter of dimension [mass], which is a nonperturbative part of a constant entering the perimeter law of the small-sized Wilson loop of a produced pair. The critical radius (i.e. such a radius, that all holes with $R < R_c$ collapse, while those with $R > R_c$ expand and destroy the pellicle) stems therefore from extremization of the action $S(R) \equiv (m+\mu)\cdot 2\pi R - \sigma\cdot\pi R^2$. This critical radius and the action of a critical hole are $R_c = (m+\mu)/\sigma$ and $S(R_c) = \pi(m+\mu)^2/\sigma$. Accordingly, in this approach, the rate of the pair production is proportional to $\mathrm{e}^{-\pi(m+\mu)^2/\sigma}$. The proportionality coefficient [16], $\frac{\sigma}{2\pi}$, should furthermore be multiplied by the factor N_f:[9]

$$w = N_f \frac{\sigma}{2\pi} \exp\left[-\frac{\pi m^2}{\sigma}\left(1+\frac{\mu}{m}\right)^2\right]. \tag{7.1}$$

Next, to evaluate both μ and σ within the same model, it is natural to use the stochastic vacuum model. It yields the following expression for the Wilson loop $\langle W(C)\rangle_{\rm YM} \equiv \frac{1}{N_c}\langle {\rm tr}\,\mathcal{P}\exp(ig\oint_C dx_\mu A_\mu)\rangle_{\rm YM}$:

$$\langle W(C)\rangle_{\rm YM} \simeq \frac{1}{N_c}\,{\rm tr}\exp\left[-\frac{1}{2!}\frac{g^2}{4}\int_{\Sigma(C)} d\sigma_{\mu\nu}(x)\int_{\Sigma(C)} d\sigma_{\lambda\rho}(x')\,\langle F_{\mu\nu}(x)\Phi_{xx'}F_{\lambda\rho}(x')\Phi_{x'x}\rangle_{\rm YM}\right], \tag{7.2}$$

where $\Sigma(C)$ is the surface encircled by the flat contour C. Next, $A_\mu \equiv A_\mu^a T^a$, where T^a's stand for the generators of the group ${\rm SU}(N_c)$ in the fundamental representation, $[T^a, T^b] = if^{abc}T^c$, ${\rm tr}\,T^aT^b = \frac{1}{2}\delta^{ab}$; the average $\langle\ldots\rangle_{\rm YM}$ is implied with respect to the euclidean Yang-Mills action, $\frac{1}{4}\int d^4x (F_{\mu\nu}^a)^2$, where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$, $a = 1,\ldots,N_c^2-1$; $\Phi_{xx'} \equiv \frac{1}{N_c}\mathcal{P}\exp(ig\int_{x'}^x dz_\mu A_\mu(z))$ is a phase factor along the straight line, which goes through x' and x. The symbol "$\simeq$" in eq. (7.2) is implied in the sense of the bilocal approximation to the cumulant expansion. This approximation, supported by the lattice data [18, 19, 20], states that, in the Yang-Mills theory, the two-point irreducible gauge-invariant correlation function (cumulant) of $F_{\mu\nu}$'s dominates over all cumulants of higher orders, which are therefore neglected. Finally, the factor $1/2!$ in eq. (7.2) is simply due to the cumulant expansion, whereas the factor $1/4$ is due to the non-abelian Stokes' theorem.

The stochastic vacuum model suggests further the following parametrization of the two-point cumulant:

$$\left\langle F_{\mu\nu}(x)\Phi_{xx'}F_{\lambda\rho}(x')\Phi_{x'x}\right\rangle_{\rm YM} =$$
$$= \frac{\hat{1}_{N_c\times N_c}}{N_c}\mathcal{N}\Big\{(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda})D\left((x-x')^2\right) +$$

[9] The factor N_c is now absent, since the Wilson loop, describing a produced pair, is a colorless object. Instead, the number of Wilson loops, which can potentially be created, is proportional to the number of different quark species, i.e. to N_f.

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$$+\frac{1}{2}\left[\partial_\mu^x\left((x-x')_\lambda\delta_{\nu\rho}-(x-x')_\rho\delta_{\nu\lambda}\right)+\partial_\nu^x\left((x-x')_\rho\delta_{\mu\lambda}-(x-x')_\lambda\delta_{\mu\rho}\right)\right]\times$$
$$\times\left[D_1\left((x-x')^2\right)+\frac{N_cC_2}{\mathcal{N}\pi^2|x-x'|^4}\right]\Bigg\}. \tag{7.3}$$

Here $C_2 \equiv \frac{N_c^2-1}{2N_c}$ is the quadratic Casimir operator of the fundamental representation, and $\mathcal{N}$ is the normalization constant, which in 4d reads $\mathcal{N} = \frac{\langle (F_{\mu\nu}^a)^2\rangle_{\rm YM}}{24[D(0)+D_1(0)]}$ [10]. Inserting this parametrization into eq. (7.2) we obtain (applying abelian Stokes' theorem to the part containing derivatives):

$$\langle W(C)\rangle_{\rm YM} \simeq \exp\Bigg\{-\frac{g^2\mathcal{N}}{8N_c}\Bigg[2\int_{\Sigma(C)} d\sigma_{\mu\nu}(x)\int_{\Sigma(C)} d\sigma_{\mu\nu}(x')D\left((x-x')^2\right)+$$
$$+\oint_C dx_\mu\oint_C dx'_\mu\left[G\left((x-x')^2\right)+\frac{N_cC_2}{\mathcal{N}\pi^2}\frac{1}{(x-x')^2}\right]\Bigg]\Bigg\}, \tag{7.4}$$

where $G(x^2) \equiv \int_{x^2}^\infty dt D_1(t)$. The one-gluon-exchange contribution to the Wilson loop, $\exp\left[-\frac{g^2C_2}{8\pi^2}\oint_C dx_\mu\oint_C dx'_\mu\frac{1}{(x-x')^2}\right] \simeq \exp(-\frac{g^2C_2}{8\pi a})$ is known [21] to yield the renormalization of mass of a produced pair. Here a stands for an inverse UV cutoff; $a \ll T_g$, and $T_g \simeq 1\,\mathrm{GeV}^{-1}$ is the correlation length of the vacuum [17]–[20]. The asymptotics of the Wilson loop at $\sqrt{|\Sigma(C)|} \gg T_g$, where $|\Sigma(C)|$ is the area of $\Sigma(C)$, is $\langle W(C)\rangle_{\rm YM} \simeq \mathrm{e}^{-\sigma|\Sigma(C)|}$. This asymptotics is obeyed by the Wilson loop of an external $q\bar{q}$-pair. Instead, at $\sqrt{|\Sigma(C)|} = \mathcal{O}(T_g)$, i.e. for the Wilson loop of a produced pair, $\langle W(C)\rangle_{\rm YM} \simeq \mathrm{e}^{-(m+\mu)L(C)}$, where $L(C)$ is the length of C.[11] As follows from eq. (7.4), the parametrization (7.3) is chosen in such a way that the function D yields the area law, while the term with the function D_1 contributes to the perimeter law. Namely, the string tension reads [24] $\sigma = \frac{g^2\mathcal{N}}{2N_c}\int d^2xD(\mathbf{x}^2)$, while for the perimeter constant μ we obtain in a way similar to the Coulomb [21] interaction between points lying on C: $\mu = \frac{g^2\mathcal{N}}{8N_c}\int_0^\infty d\xi G(\xi^2)$. A derivation of this formula is presented in appendix A.

In what follows, we will adopt the exponential parametrization of the functions D and D_1 [18]–[20] $D(x^2) = D(0)\mathrm{e}^{-|x|/T_g}$, $D_1(x^2) = D_1(0)\mathrm{e}^{-|x|/T_g}$. It yields the following values of the string tension and the perimeter constant: $\sigma = \frac{\pi g^2\mathcal{N}}{N_c}T_g^2D(0)$, $\mu = \frac{g^2\mathcal{N}}{2N_c}T_g^3D_1(0)$. Introducing the parameter $\gamma \equiv D_1(0)/D(0)$, whose lattice value in full QCD is 0.13 ± 0.08 (see e.g. [22]), we can rewrite the obtained results as

$$\sigma = \frac{\pi}{24(1+\gamma)N_c}g^2\left\langle(F_{\mu\nu}^a)^2\right\rangle T_g^2\,, \qquad \mu = \frac{\gamma}{48(1+\gamma)N_c}g^2\left\langle(F_{\mu\nu}^a)^2\right\rangle T_g^3\,. \tag{7.5}$$

For the critical radius of a hole we have $R_c = \frac{m+\mu}{\sigma} = \frac{m}{\sigma} + \frac{\gamma}{2\pi}T_g$. Substituting again $\sigma = (440\,\mathrm{MeV})^2$, $m = 200\,\mathrm{MeV}$, and $T_g^{-1} = 1\,\mathrm{GeV}$, we get for the worst case of the above-quoted lattice value of γ, $\gamma = 0.21$, $\frac{R_c}{T_g} \simeq 1.07$.[12] This ratio is certainly $\mathcal{O}(1)$, that justifies the use of the perimeter asymptotics for the Wilson loop of a produced pair.

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[10] Note that the one-gluon-exchange contribution, represented by the $\frac{1}{|x-x'|^4}$-term, can alternatively be considered as a part of the function D_1 [18]. Here, we rather consider this perturbative contribution separately, so that $D_1(0)$ is a finite quantity.

[11] Below, we will see that the typical size of the Wilson loop of a produced pair is indeed $\mathcal{O}(T_g)$.

[12] Notice that T_g itself is smaller (by a factor of the order of 5) than a typical size of the Wilson loop of an external $q\bar{q}$-pair, at which the onset of string-breaking is normally expected.

Comparing the pair production rate, eq. (7.1), with eq. (2.10), we see that the parameter $\left(1+\frac{\mu}{m}\right)^2$ replaces the parameter $L/4$ of that equation. At $\sigma = (440\,\mathrm{MeV})^2$, $m = 200\,\mathrm{MeV}$, we have[13] $(1+\frac{\mu}{m})^2 \simeq 1.08$, whereas, according to eq. (2.9), $\frac{L}{4} < 1.55$, that is of the same order of magnitude. Finally, the new string-breaking distance, at which the potential $V(r) = \sigma r \mathrm{e}^{-\pi r^2 \cdot w}$ acquires its maximum, is

$$\bar{r} = \frac{1}{\sqrt{2\pi w}} = \frac{1}{\sqrt{N_f \sigma}} \exp\left[\frac{\pi m^2}{\sigma}\left(1+\frac{\mu}{m}\right)^2\right]. \tag{7.6}$$

The argument of the exponent here approximately equals 0.70, that is quite similar to 0.5 we had as an upper boundary in case of eq. (2.11). However, contrary to that equation, eq. (7.6) does not contain the factor $\sqrt{L}$ in the preexponent.

Notice also that, due to the fact that $\sigma = \mathcal{O}(N_c^0)$, the string-breaking distance (2.11) is $\mathcal{O}(N_c^0)$ too [6]. For the same reason, as can be seen from the first of eqs. (7.5), T_g is $\mathcal{O}(N_c^0)$ as well. Therefore, according to the second equation of (7.5), μ is also $\mathcal{O}(N_c^0)$. This means that, as well as eq. (2.11), the new string-breaking distance (7.6) is $\mathcal{O}(N_c^0)$.

8. Summary

In this paper, we have considered two approaches to the problem of string breaking in QCD: one of these is based on the dual superconductor model of confinement, and the other one — on the stochastic vacuum model. In the first approach, which was proposed already in ref. [6], the pair-production mechanism is due to the field of the chromoelectric flux tube (dual Abrikosov-Nielsen-Olesen string in the London limit of the dual abelian Higgs model). In the second approach, pairs are considered as holes in a pellicle, which confines a test $q\bar{q}$-pair.

Within the first approach, we have demonstrated that the result of ref. [6], based on the Schwinger formula, accounts only for the first term of the cumulant expansion in the average (4.1). In section 3, we have found the temperature dependences of the string-breaking distance at temperatures close to the critical one and at low temperatures, smaller than $\mathcal{O}(f_\pi)$. In both cases, the string-breaking distance increases with the increase of the temperature. We have also noticed that, in the case of spin-$\frac{1}{2}$ quarks, antiperiodic boundary conditions for fermions produce only corrections which fall off as the tail of the gaussian distribution, as long as $T < m_\pi/\sqrt{2}$. As a by-product, we have found in appendix B the temperature dependence of the string tension, which reproduces correctly the zero-temperature value. In section 4, we have calculated (for scalar quarks) a correction, generated by the second cumulant in the expansion (4.1), i.e. by the dispersion of the chromoelectric field in the direction transverse to the string. This effect slightly diminishes the string-breaking distance. Using the Feynman variational method, we have then derived in section 4 the leading quantum correction to this effect, produced by the deviation of the trajectory of a pair from the classical one. This effect further diminishes the string-breaking distance.

[13] The correction $\frac{\mu}{m}$ is very small: $\frac{\mu}{m} = \frac{\gamma}{2\pi}\frac{T_g \sigma}{m} \simeq 0.02$.

The effects of dispersion of the chromoelectric field are small as long as the cumulant expansion is convergent, that is the case when the logarithm of the Landau-Ginzburg parameter is bounded from above according to (4.6). This constraint is, however, always obeyed as long as at least the first pole in the Schwinger formula gives its contribution, that leads to an even more severe constraint (2.9). Although both constraints have been shown to relax at $T \to T_c$, they do exist at $T = 0$. This fact necessitates to perform the average (4.1) without the use of the cumulant expansion at all, that has been done in section 6. As a result, the upper boundary on the logarithm of the Landau-Ginzburg parameter increases by a factor 4 (cf. eq. (6.1)). Furthermore, a novel formula (6.3) for the rate of the pair production has been derived. At $T = 0$, some range of the Landau-Ginzburg parameter has been found, in which the string-breaking distance is larger than the one we had with the use of the bilocal approximation to the cumulant expansion by a factor varying between 2.84 and 9.10.[14] Instead, at $T \to T_c$, this factor is smaller, namely it equals $\sqrt{2}$. Notice that such an analytic average of the exponent (4.1) without the use of the cumulant expansion was only possible due to the explicit form of the Abrikosov-Nielsen-Olesen solution in the London limit. Apparently, studies away from this limit will require a numerical analysis.

Within the approach, which treats pairs as holes in the confining pellicle, the new quantity on which the string-breaking distance is dependent is the constant entering the perimeter law of the Wilson loop of a produced pair. For typical values of the hadronic mass, string tension, and the vacuum correlation length, this dependence is, however, very weak. As for the dependence of the new expression for the string-breaking distance on the mass of a produced pair and on the string tension, it is the same as in the above-discussed case of the approach based on the Schwinger formula, where only the second cumulant is taken into account. Finally, the results for the string-breaking distance, obtained within all the above-mentioned approaches, are $\mathcal{O}(N_c^0)$ in the large-N_c limit.

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A. Some technical details

Let us first present evaluation of the dispersion parameter (4.5). Integrating both sides of the equality $\frac{1}{2\pi}K_0(mr) = \int \frac{d^2p}{(2\pi)^2}\frac{e^{i\mathbf{p}\mathbf{x}_\perp}}{p^2+m^2}$ over d^2r we trivially have $\int d^2r K_0(mr) =$

[14]This result indicates that, at least at these values of the Landau-Ginzburg parameter, the effect produced by cumulants higher than the quadratic one is opposite and stronger than the result produced by the quadratic cumulant alone.

$\int \frac{d^2p}{p^2+m^2}\delta(\mathbf{p}) = \frac{1}{m^2}$. In the same way

$$\int d^2rK_0^2(mr) = \int d^2r \int \frac{d^2pd^2q}{(2\pi)^2} \frac{\mathrm{e}^{i(\mathbf{p}+\mathbf{q})\mathbf{x}_\perp}}{(p^2+m^2)(q^2+m^2)} = \frac{1}{2\pi}\int \frac{d^2p}{(p^2+m^2)^2} = \frac{1}{2m^2}\,. \tag{A.1}$$

The parameter (4.5) then reads

$$d = \frac{\int d^2rK_0^2(m_Vr) - \frac{m_V^2}{\pi}\left[\int d^2rK_0(m_Vr)\right]^2}{\frac{m_V^2}{\pi}\left[\int d^2rK_0(m_Vr)\right]^2} = \frac{\frac{1}{2m_V^2} - \frac{1}{\pi m_V^2}}{\frac{1}{\pi m_V^2}} = \frac{\pi}{2} - 1\,.$$

Next, we will discuss some details of derivation of c_{var}. Taylor expanding $K_0(z')$ in eq. (5.3) to the second order we have

$$\begin{aligned}
&\frac{1}{m_V^4}\int d^2zd^2z'\mathrm{e}^{-\frac{3}{2Tm_V^2}(\mathbf{z}-\mathbf{z}')^2}K_0(z)K_0(z') \simeq \\
&\simeq \frac{2\pi T}{3m_V^2}\int d^2zK_0^2(z) - \frac{1}{m_V^4}\int d^2z\frac{z_\mu}{z}K_1(z)\int d^2z'(z'-z)_\mu\mathrm{e}^{-\frac{3}{2Tm_V^2}(\mathbf{z}-\mathbf{z}')^2} + \\
&+ \frac{1}{2m_V^4}\int d^2z\left[\frac{z_\mu z_\nu - z^2\delta_{\mu\nu}}{z^3}K_1(z) + \frac{z_\mu z_\nu}{2z^2}(K_0(z)+K_2(z))\right]K_0(z)\times \\
&\times \int d^2z'(z'-z)_\mu(z'-z)_\nu\mathrm{e}^{-\frac{3}{2Tm_V^2}(\mathbf{z}-\mathbf{z}')^2}\,.
\end{aligned}$$

The second term on the r.h.s. of this equation apparently vanishes, while the third ones reads $\frac{\pi T^2}{9}\int d^2zK_0[\frac{1}{2}(K_0+K_2) - \frac{1}{z}K_1]$. Using the definition $c_{\mathrm{var}} \equiv \frac{g^2}{8}(\langle\mathcal{E}^2\rangle - \langle E\rangle^2)$, we then arrive at the following intermediate result:

$$c_{\mathrm{var}} = c + \frac{\pi\sigma^3T}{3g^2L^3}\int_0^\infty dzK_0\left[z(K_0+K_2) - 2K_1\right].$$

Here, the addendum c, eq. (4.2), is apparently produced by the term with no derivatives, while the other addendum is produced by the second-derivative term of the Taylor expansion. Finally, the integral over z can be calculated exactly. It reads

$$\frac{1}{2\pi}\int d^2zK_0^2 + \int_0^\infty dzK_0\left[z\left(K_0 + \frac{2}{z}K_1\right) - 2K_1\right] = \frac{1}{\pi}\int d^2zK_0^2 = \frac{1}{2\pi}\,,$$

where eq. (A.1) on the last step has been used. This yields eq. (5.4).

We will finally present a proof of the formula

$$\oint_C dx_\mu \oint_C dx'_\mu G\left((x-x')^2\right) \simeq L\cdot\int_0^\infty d\xi G(\xi^2)\,, \tag{A.2}$$

where $L = \int_0^1 ds|\dot{x}_\mu(s)|$ is the length of the contour C. Since the function $G(x^2)$ is rapidly decreasing (e.g. $G(x^2) = 2D_1(0)T_g(|x|+T_g)\mathrm{e}^{-|x|/T_g}$ for the adopted Ansatz $D_1(x^2) = D_1(0)\mathrm{e}^{-|x|/T_g}$), we can Taylor expand $x'_\mu \equiv x_\mu(s+t)$ as $x_\mu(s+t) \simeq x_\mu(s) + t\dot{x}_\mu(s)$. In the expression under study, $\int_0^1 ds\int_0^1 dt\dot{x}_\mu(s)\dot{x}_\mu(s+t)G(t^2\dot{x}_\mu^2(s))$, we therefore have $\dot{x}_\mu(s)\dot{x}_\mu(s+t) \simeq \dot{x}_\mu^2(s) + t\dot{x}_\mu(s)\ddot{x}_\mu(s) = \dot{x}_\mu^2(s)$, where, at the last step, the proper-length parametrization $\dot{x}_\mu^2 = \mathrm{const}$ has been fixed. Introducing instead of t the new integration variable $\xi = t|\dot{x}_\mu|$, we have for the expression under study $\int_0^1 ds|\dot{x}_\mu(s)|\int_0^{|\dot{x}_\mu|} d\xi G(\xi^2)$. Finally, due to the rapid decrease of $G(\xi^2)$, the upper integration limit in the last integral can be replaced by infinity, that completes the proof of eq. (A.2).

B. Temperature dependence of the string tension

To find the dependence $\sigma(T)$, notice that the string tension can be derived from the string representation of the partition function [23]. An essential result of this representation is the following string effective action:

$$2(\pi v)^2 \int d\sigma_{\mu\nu}(x) \int d\sigma_{\mu\nu}(x') D_{m_V}(x-x')\,,$$

where $D_m(x) \equiv mK_1(m|x|)/(4\pi^2|x|)$ is the Yukawa propagator. The zero-temperature string tension then stems from this action according to the general (for this type of non-local string actions) formula [24]: $\sigma_0 = (\pi v)^2 \cdot \frac{4}{m_V^2} \int_{m_V/m_H} d^2z \frac{m_V^2 K_1(|z|)}{4\pi^2|z|}$. This indeed equals the above-used value $2\pi v^2 L$ (following from the Landau-Ginzburg equations). At finite temperatures (smaller than the temperature of dimensional reduction), we rather have the following expression in terms of Matsubara frequencies:

$$\sigma(T) = v^2 \int_{m_V/m_H} d^2z \sum_{n=-\infty}^{+\infty} \frac{K_1\left(\sqrt{z^2+(m_V\beta n)^2}\right)}{\sqrt{z^2+(m_V\beta n)^2}}\,.$$

To perform the integration, it is useful to transform the sum as follows:

$$\sum_{n=-\infty}^{+\infty} \frac{K_1\left(\sqrt{z^2+(m_V\beta n)^2}\right)}{\sqrt{z^2+(m_V\beta n)^2}} = \sum_{n=-\infty}^{+\infty} \int_0^\infty dt \exp\left[-\frac{1}{4t} - t\left(z^2+(m_V\beta n)^2\right)\right]$$
$$= \frac{T\sqrt{\pi}}{m_V} \sum_{n=-\infty}^{+\infty} \int_0^\infty \frac{dt}{\sqrt{t}} \exp\left[-z^2 t - \frac{1+(\omega_n/m_V)^2}{4t}\right] = \frac{\pi T}{m_V|z|} \sum_{n=-\infty}^{+\infty} \mathrm{e}^{-|z|\sqrt{1+(\omega_n/m_V)^2}}\,.$$

Integration of this expression over d^2z is now trivial and yields

$$\sigma(T) = \frac{2\pi^2 T v^2}{m_V} \sum_{n=-\infty}^{+\infty} \frac{\mathrm{e}^{-\frac{m_V}{m_H}\sqrt{1+(\omega_n/m_V)^2}}}{\sqrt{1+(\omega_n/m_V)^2}}\,.$$

Performing now with the sum a transformation of the form (3.3) in the opposite direction, we obtain

$$\sigma(T) = 2\pi v^2 \sum_{n=-\infty}^{+\infty} K_0\left(\frac{m_V}{m_H}\sqrt{1+(m_H\beta n)^2}\right).$$

In particular, at $T \to 0$, one may approximate the sum by the zeroth term, that recovers the value $\sigma_0 = 2\pi v^2 L$.

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